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XV. *On the Extraction of Roots. By* BENJAMIN WEST, *Eſquire, F. A. A. In a Letter to Mr.* CALEB GANNETT, *Rec. Sec. F. A. A.*

DEAR SIR,

HAVING a little leiſure at this time, I thought it a proper opportunity to fulfil my promiſe of ſending you the ſubſtance of what I have done, reſpecting the Roots. What I chiefly aimed at was, to render the method of extracting the roots of the odd powers eaſier, and leſs burthenſome to the memory; and, I think, I have not failed in my attempt. The method, followed by *Ward* and others, is excellent, but is attended with too much difficulty in getting the diviſors; eſpecially for learners, who are not acquainted with the reaſon of the rules.—That difficulty I have ſtrove to remedy in the following work. In the courſe of my teaching, ſince I made this diſcovery, I have not met with more trouble in teaching the third, fifth and ſeventh powers, than what is uſual in extracting the ſquare root. Methods, ſimilar to theſe, may be found for extracting the roots of the even powers.

I generally get the three firſt figures in the root by the firſt operation, (if there are ſo many in the root) tho' many times four or five may be got, if the firſt figure in the root comes very near to the firſt period of the reſolvend.

As it is not neceſſary to make a long introduction, I ſhall proceed to ſhow in what manner I inveſtigated the rules. The cube root being the firſt in order, I ſhall begin with that; and in order thereto, let the reſolvend be $= a$; the firſt figure in the root $= r$; and the remaining figures in the root $= e$; and then the root will be $r+e$; and, by involution $r^3+3r^2e+3re^2+eee=a$. By dropping e^3, and making the other terms equal to a, we ſhall have $r^3+3re^2+3re=a$; and by ſubtraction $3re^2+3re^2=a-r^3$; and by diviſion $e^2+re=\frac{a-r^3}{3r}$; and by compleating the ſquare, we ſhall have $e^2+re+\frac{r^2}{4}=\frac{a-r^3}{3r}+\frac{r^2}{4}=\frac{4a-r^3}{12r}$; by extracting the ſquare root of both ſides, it will give $e+\frac{r}{2}=\sqrt{\frac{4a-r^3}{12r}}$; whence $e=-\frac{r}{2}+\sqrt{\frac{4a-r^3}{12r}}$; and by the addition of r on each ſide, it will be $r+e=r-\frac{r}{2}+\sqrt{\frac{4a-r^3}{12r}}=\frac{r}{2}+\sqrt{\frac{4a-r^3}{12r}}$. From which comes this rule :

Take the neareſt root to the firſt period of the reſolvend, be it more or leſs than juſt, and ſupply it with as many cyphers as there are remaining periods in the reſolvend, and call it the aſſumed root. Then multiply the given reſolvend by 4; from the product ſubtract the cube of the aſſumed root; divide the remainder by 12 times the aſſumed root, and extract the ſquare root of the quotient. To the ſquare root, thus found, add one half of the aſſumed root, and it will give the cube root corrected; which may be repeated and carried to what exactneſs you may wiſh to have it;—always taking the laſt root for the aſſumed root.

Take an example or two.—Let it be required to extract the cube root from $73\dot{5}05\dot{1}27\dot{4}$. Here 9 is the neareſt root 735, and 900 is the aſſumed root, leſs than juſt.

```
               735051274(900=aſſumed root.
                      ×4
               ------------
      3         2940205096
   900= —       729000000
               ------------
900×12=108|00)22112050|96(204742
               216......
               -----
                512            4   )    204741(452,
                432            4        16        450=the aſſumed root;
                ----           --       ----      ----
                 800           85        447      902=true root, nearly.
                 756            5        425
                 ----          ---       ----
                  445          902        2241
                  432                     1804
                  ----                    ----
                   130                     437
                   108
                   ---
                    22
```

Take a ſecond example. Extract the cube root from 330625.

```
         330625(70=aſſumed root, more than juſt.
              ×4
         ---------                          The work repeated,
   3      1322500
  70 = —   343000                                 69,1
         ---------                                69,1
70×12=84|0)9795|0|0  (  1166,07 (34,1             ----
          84...   3     9        35,=½70          691
          ----    3     ---      -------         6219
          139    ---    266      69,1 corrected. 4146
           84    64     256                      ---------
          ----    4     ----                     4774,81 ſquare.
           555   ---     1007                       69,1
           504   681      681                    ---------
           ----           ----                   2864886
            510            326                    429733
            504                                     4774
            ----                                 ---------
             600                                 329939,3 cube.
             588
             ----
              12
```

$$69,1^3 = \begin{array}{r} 1322500 \\ -\ 329939,3 \\ \hline \end{array}$$

$69,1 \times 12 = 829,2)992560,7(1197,010009647852$

8292

16336
8292

80440
74628

58127
58044

8300
8292

80000
74628

53720
49752

39680
33168

65120
58044

70760
66336

4324
4146

178
165

13

3) 1197,010009647852(34,597832

3	9
64	297
4	256
685	4101
5	3425
6909	67600
9	62181
69187	541909
7	484309
691948	5760064
8	5535584
6919563	22448078
3	20758689
69195662	168938952
	138391324
	30547628

$34,55 = \frac{1}{2}69,1$

69,147832 root.

Two examples being ſufficient to illuſtrate the rule, I ſhall proceed to the ſurſolid, or fifth power. The inveſtigation is as follows :

Let a = the reſolvend ; r = the firſt figure in the root, and e = the remaining figures. Then $r + e$ = true root, and $r^5 + 5r^4e + 10r^3e^2 + 10r^2e^3 + 5re^4 + e^5 = a$; and by dropping all the terms that contain a higher power of e than e^2, and by by making the remainder equal to a, there will be $r^5 + 5r^4e + 10r^3e^2 = a$, and then $5r^4e + 10r^3e^2 = a - r^5$; and by diviſion $e^2 + \frac{1}{2}re = \frac{a-r^5}{10r^3}$; and by compleating the ſquare we ſhall have $e^2 + \frac{1}{2}re + \frac{r^2}{16} = \frac{a-r^5}{10r^3} + \frac{r^2}{16} = \frac{16a-6r^5}{160r^3} = \frac{8a-3r^5}{80r^3}$*; and by extracting the ſquare root, $e + \frac{r}{4} = \sqrt{\frac{8a-3r^5}{80r^3}}$; and by ſubtraction $e = -\frac{r}{4} + \sqrt{\frac{8a-3r^5}{80r^3}}$; and by adding r on both ſides; $r + e = r - \frac{r}{4} + \sqrt{\frac{8a-3r^5}{80r^3}} = \frac{3}{4}r + \sqrt{\frac{8a-3r^5}{80r^3}}$. Whence this rule:

Take the neareſt root to the firſt period of the reſolvend, more or leſs than juſt, and annex as many cyphers, as there are remaining periods in the reſolvend, and call it the aſſumed root. Multiply the reſolvend by 8, and from the product take 3 times the fifth power of the aſſumed root; divide the remainder by 80 times the third power of the aſſumed root, and extract the ſquare root of the quotient; to which add three fourths of the aſſumed root, and it will give the ſurſolid root corrected:—It may be repeated, and carried to what exactneſs you would have it; ſtill taking the laſt root from the aſſumed root.

Let us take an example from *Ward:* Extract the ſurſolid root from 12309502009375. Here the neareſt ſurſolid root to 1230 is 4, and the aſſumed root 400, leſs than juſt.

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* The co-efficients are divided by their greateſt common meaſure. *Euc.* b. 7. p. 3.

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                          ·   ·  ·
                    12309502009375 (400=aſſumed root.
                                x8
                    ──────────────
          5         98476016075000
   400x3=—          30720000000000
   3                ──────────────            · · ·
400x80=512 0000000)6775601|6075000 (   13233 ( 115
                    512····                1       300
                    ────           1       ──      ────
                    1655           1       32      415 = root.
                    1536           ─       21
                    ─────          21      ────
                     1196           1      1133
                     1024          ───     1125
                     ─────         225     ────
                      1720                    8
                      1536
                      ─────
                       1841
                       1536
                       ────
                        305
```

One caſe being ſufficient to exemplify the rule, I ſhall not trouble you with a ſecond. The next is the ſecond ſurſolid, or ſeventh power: And let the ſame letters ſtand for the ſame things, as in the preceding powers; and then we ſhall have $r^7 + 7r^6e + 21r^5e^2 + 35r^4e^3 + 35r^3e^4 + 21r^2e^5 + 7re^6 + e^7 = a$; and by neglecting all the terms that contain a greater power of e than e^2, and making the remainder equal to a, there will ariſe $r^7 + 7r^6e + 21r^5e^2 = a$; and $7r^6e + 21r^5e^2 = a - r^7$; and by diviſion $e^2 + \frac{1}{3}re = \frac{a-r^7}{21r^5}$; and by compleating the ſquare, $e^2 + \frac{1}{3}re + \frac{r^2}{36} = \frac{a-r^7}{21r^5} + \frac{r^2}{36} = \frac{12a-5r^7}{252r^5}$:* By extracting the ſquare root of both ſides of the equation, there will ariſe $e + \frac{1}{6}r = \sqrt{\frac{12a-5r^7}{252r^5}}$; and $e = -\frac{1}{6}r + \sqrt{\frac{12a-5r^7}{252r^5}}$. And furthermore, if r be added to both ſides of the equation, there will be had, $r + e = r - \frac{1}{6}r + \sqrt{\frac{12a-5r^7}{252r^5}} = \frac{5}{6}r + \sqrt{\frac{12a-5r^7}{252r^5}}$. Whence this rule:

Take

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* See note, p. 166.

Take the neareſt ſecond ſurſolid root to the firſt period of the reſolvend, and place as many cyphers at it's right hand as there are remaining periods in the reſolvend, and call it the aſſumed root. Multiply the reſolvend by 12, and take 5 times the ſeventh power of the aſſumed root from the product, and divide the remainder by 252 times the fifth power of the aſſumed root; extract the ſquare root of the quotient, and add the root to $\frac{5}{6}$ of the aſſumed root, and it will give the ſecond ſurſolid root corrected; which may be repeated and carried to the deſired accuracy,—remembering to take, in each operation, the laſt root for the aſſumed root. We will take *Ward's* example, viz. Extract the ſecond ſurſolid root from $38\dot{2}9865539\dot{5}5078125$ Here the neareſt root to 3829, is 3, and 300 is the aſſumed root, leſs than juſt.

$38\dot{2}98655395\dot{5}078125\dot{5}$ (300

$\times 12$

4595838647460937500

$300^7 \times 5 = $ — 10350000000000000000

$300^5 \times 252 = 61236$)0000000000)350233864|7460937500($5\dot{7}1\dot{9}$

306180...

440538

428652

118866

61236

576304

541124

35180

7) $\dot{5}7\dot{1}9$ (75

7   49   250 = $\frac{5}{6}$ of the aſſumed root.

145   819   325 = root, agreeable to *Ward.*

725

94

SCHOLIUM.

It may be obſerved, that in the foregoing examples, I have carried the work out at large : This I did in order to avoid any ſeeming embarraſſments. Whatever contractions are made uſe of by *Ward*, or others, may be adopted, with equal propriety, in theſe methods ; and I have not met with an inſtance,in which theſe rules do not approach the root with equal celerity, as thoſe of the forementioned author.

I am, &c.

BENJAMIN WEST.

To Mr. *Caleb Gannett*.

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